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Ground States of Quantum Spin Systems

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Abstract. We prove that ground states of quantum spin systems are characterized by a principle of minimum local energy and that translationally invariant ground states are characterized by the principle of minimum energy per unit volume.

I. Introduction

Let τ be a strongly continuous one-parameter group of *-automorphisms of a C^*-algebra $\mathfrak{A}$. Denote the generator of τ by δ. A state ω over $\mathfrak{A}$ is defined to be a τ-ground state if

$$-i\omega(A^*\delta(A))\geqq 0$$

for all A in the domain $D(\delta)$ of δ. It follows that ω is τ-invariant and hence generates a covariant representation $(\mathscr{H}_\omega, \Pi_\omega, U_\omega, \Omega_\omega)$ of $(\mathfrak{A}, \tau)$. If U_ω is chosen such that $U_\omega(t)\Omega_\omega = \Omega_\omega$ and if H_ω is the infinitesimal generator of U_ω then the ground state condition implies $H_\omega \geqq 0$. More generally a state ω is a τ-ground state if, and only if, ω is τ-invariant and $H_\omega \geqq 0$. (For further details see [1], Chapter V).

A state ω is called a (τ, β)-KMS state, for $\beta \in \mathbb{R}$, if

$$\omega(AB) = \omega(B\tau_{i\beta}(A))$$

for all entire analytic elements A, B of τ. The (τ, β)-KMS states arise from the Gibbs formalism of equilibrium statistical mechanics and correspond to equilibrium states at inverse temperature β. The ground states correspond to the zero temperature states.

One can prove, for example by the Sewell condition [2] (see also [3]) characterizing (τ, β)-KMS states, that if ω_β is a family of (τ, β)-KMS states which is weak*-convergent as $\beta \to \infty$ then the limit state ω is a τ-ground state. For this reason ground states are sometimes referred to as $(\tau, +\infty)$-KMS states. This

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