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# Occurrence of Strange Axiom $A$ Attractors Near Quasi Periodic Flows on $T^m$, $m \geqq 3$

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**Abstract.** It is shown that by a small $C^2$ (resp. $C^\infty$) perturbation of a quasi-periodic flow on the 3-torus (resp. the $m$-torus, $m > 3$), one can produce strange Axiom $A$ attractors. Ancillary results and physical interpretation are also discussed.

## 1. Statement of Results

The main purpose of this note is to prove the following fact.

**Theorem 1.** *Let $a = (a_1, \ldots, a_n)$ be a constant vector field on the torus $T^n = \mathbb{R}^n/\mathbb{Z}^n$.*

*If $n = 3$, in every $C^2$ neighborhood of $a$ there is a vector field satisfying Axiom A and having a non trivial attractor.*

*If $n \geqq 4$, in every $C^\infty$ neighborhood of $a$ there is a vector field satisfying Axiom A and having a non trivial attractor.*

We say that an Axiom $A$ attractor is non trivial (or "strange") if it does not consist of a single periodic orbit (for general definitions, see Smale [7]). The above theorem improves a result of Ruelle and Takens [6], and can be obtained by simple modifications of the proof given there. We nevertheless give here a complete proof, based on the following result which is of interest in itself.

**Theorem 2.** *Let $M$ be a $C^\infty$ compact manifold of dimension $m$.*

(a) *If $m = 2$, in every $C^1$ neighborhood of the identity there is an Axiom A diffeomorphism with a non trivial attractor.*

(b) *If $M = T^2$, in every $C^2$ neighborhood of the identity there is an Axiom A diffeomorphism with a non trivial attractor.*

(c) *If $m \geqq 3$, in every $C^\infty$ neighborhood of the identity there is an Axiom A diffeomorphism with a non trivial attractor.*

The proof of these theorems is given in Sect. 2. In Sect. 3 we discuss non trivial attractors for Axiom $A$ diffeomorphisms of two-dimensional manifolds. Section 4 is devoted to physical interpretation of Theorem 1.

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