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# Pointwise Bounds on Eigenfunctions and Wave Packets in $N$-Body Quantum Systems IV*

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**Abstract.** We describe several new techniques for obtaining detailed information on the exponential falloff of discrete eigenfunctions of $N$-body Schrödinger operators. An example of a new result is the bound (conjectured by Morgan) $|\psi(x_1 \dots x_N)| \leqq C \exp(-\sum_1^N \alpha_n r_n)$ for an eigenfunction $\psi$ of

$$H_N = -\sum_{i=1}^{N} \left(\Delta_i - \frac{Z}{|x_i|}\right) + \sum_{i<j} |x_i - x_j|^{-1}$$

with energy $E_N$. In this bound $r_1 r_2 \dots r_N$ are the radii $|x_i|$ in increasing order and the $\alpha$'s are restricted by $\alpha_n < (E_{n-1} - E_n)^{1/2}$, where $E_n$, for $n = 0, 1, \dots, N-1$, is the lowest energy of the system described by $H_n$. Our methods include subharmonic comparison theorems and "geometric spectral analysis".

## § 1. Introduction

It is an elementary fact that a solution of $(-\Delta + V)\psi = E\psi$ with $\psi \in L^2$, $V \to 0$ at $\infty$ (in some sense) and $E < 0$ has exponential falloff: it is certainly bounded (in some sense) by $C(\exp(-(1-\varepsilon)\sqrt{-E}|x|)$ for any $\varepsilon > 0$. Our interest here is in a considerably more subtle situation. Let

$$\tilde{H} = -\sum_{i=1}^{N} (2m_i)^{-1} \Delta^i + \sum_{i<j}^{1 \dots N} V_{ij}(x^i - x^j) \tag{1.1}$$

on $L^2(R^{\nu N})$ be the Schrödinger operator for $N$ particles with coordinates $x^i \in R^\nu$, where $\Delta^i$ = Laplacian with respect to $x^i$. The operator $H$ obtained by separating the center of mass motion acts on $L^2(X)$ where $X$ is the $\nu(N-1)$-dimensional subspace $\Sigma m_i x^i = 0$ of $R^{\nu N}$. (Kinematics is discussed in Appendix 1). We consider solutions of

$$H\psi = E\psi. \tag{1.2}$$

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