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Instantons and Kähler Manifolds

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Abstract. It is shown that for two-dimensional Euclidean chiral models of the field theory with values in arbitrary Kähler manifold "duality equations" reduce to the Cauchy-Riemann equations on this manifold. A class of models is described possessing such type solutions, the so called instanton solutions.

1. In the last few years a considerable progress has been achieved in studying both pseudoeuclidean and Euclidean chiral models of the field theory, i. e. the models for which the field takes the values in nonlinear manifolds (see refs. [1–7]). Note especially the recent results by V. E. Zakharov and A. V. Mikhailov who developed the method of finding the explicit solutions for a certain class of two-dimensional pseudoeuclidean chiral models [7].

In many cases the solutions of field equations can be characterized by topological invariants, the so called topological charges, which allow to estimate the energy (action) of a system from below [1, 2, 8–10]. The solutions of the Euclidean theory equations with a certain topological charge corresponding to the minimum of energy (action), the solutions of the so called "duality equations" are usually called instanton solutions. For the case when the field takes the values in the two-dimensional sphere S^2, or, that is the same, in the one-dimensional complex projective space CP^1 such a problem has been solved in the paper by Belavin and Polyakov [2] while for the case of $CP^n(n>1)$ in the paper [10].

In this work which is an elaboration of the investigation started in [10] we show that just in [2] and in [10], for the two-dimensional chiral models of the field theory with the values in arbitrary compact Kähler manifold[1] the "duality equations" reduce to the Cauchy-Riemann equations on this manifold. The class of manifolds for which such solutions do exist is described.

These are the Kähler homogeneous simply-connected manifolds. Such manifolds can be as well characterized by that they are homogeneous under the

1 For the properties of Kähler manifolds and of more general class complex manifolds as well see the book [11]

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