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Conservation Laws for Classes of Nonlinear Evolution Equations Solvable by the Spectral Transform

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Abstract. The existence of conservation laws for novel classes of nonlinear evolution equations (with linearly x-dependent coefficients) solvable by the spectral transform is investigated. A remarkably explicit representation is moreover obtained for the conserved quantities of the "old" classes of nonlinear evolution equations (with x-independent coefficients; including the Korteweg-de Vries equation, the modified Korteweg-de Vries equation, the nonlinear Schrödinger equation, etc.).

1. Introduction

A characteristic feature of the class of nonlinear evolution equations solvable via the spectral transform (see, for instance, [1–20]) is the existence of an infinite number of conserved quantities (see, for instance, [1–28]). Indeed the discovery of an infinite number of local conservation laws for the Korteweg-de Vries (KdV) and modified Korteweg-de Vries (mKdV) equations has preceeded, and paved the way, to the introduction of the inverse scattering transform (or, as we prefer to call it, the *spectral transform*); it has played a crucial rôle in certain fundamental developments of the theory of these equations, such as the hamiltonian formulation (see for instance [4–5]); and it has been the subject of many papers, including several recent ones (see, for instance, [17–19], [22–28], and the paper by Wadati in [20]).

Recently we have introduced an extension of the approach based on the spectral transform that enlarges the classes of nonlinear evolution equations solvable by this technique [29–32]. A characteristic feature of these extended classes is that they are no more associated to isospectral flows. One finds accordingly that the existence of an infinite number of local conservation laws holds no more. It is however still possible to exhibit conserved quantities, although their practical usefulness is doubtful, since each of them is a linear combination of an infinite number of the "old" conserved quantities. It is moreover possible to

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