Commun. math. Phys. 63, 31–47 (1978)

Communications in Mathematical Physics

On Confinement of Fermions in Strongly Coupled Lattice Gauge Theory

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Abstract. A lattice theory of Fermi fields of mass m coupled to gauge fields in the region where m and the gauge field coupling constant g are large is studied. It is shown that the energy of some states composed of a fermion and a distant antifermion with a string in between grows at least linearly with the distance if $1 < g^6 < m < g^{\varepsilon \log g}$.

1. Introduction

The lattice gauge field theory was formulated by Wilson [15] with the hope that the mechanisms behind the long distance behavior of the continuum fields could be understood in the simplified lattice models, see also [1, 7]. The primary aim was to understand the quark confinement. Working with the QED Wilson formulated a criterion for charge confinement which involved only the electromagnetic field (no Fermi fields):

if $\left\langle \exp\left(ie \int_{\partial s} A_\mu dx^\mu\right)\right\rangle \sim \exp(-C(s))$ and $C(s)$ is proportional to the area of the two-dimensional cube s then charge should be confined; if $C(s)$ is proportional to the circumference of s then no confinement occurs.

The criterion was based on the analysis of the expansion of Euclidean propagators of full lattice QED into powers of, say, inverse fermion mass, interpreted as a sum over fermion-antifermion trajectories. Each path σ contributed a lattice version of $\left\langle \exp\left(ie \int_\sigma A_\mu dx^\mu\right)\right\rangle$. It was argued that in the case of the "area law", paths with fermion and antifermion well separated hardly entered. Wilson suggested that in the lattice QED the area law should hold for large coupling constant g. This was confirmed by the rigorous result of Osterwalder-Seiler [10] obtained for a wide class of lattice gauge theories. In the meantime

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