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Absence of Classical Lumps in Constrained Systems

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Abstract. We prove the absence of classical lumps for a large class of constrained systems. In particular we prove that there is no classical lump in the $0(N)$ non linear σ-model in 2 dimensional space time.

I. Introduction

The possibility of describing particles by quantization of classical lumps[1] (i.e. finite energy classical solutions to the field equations in Minkowski space having the property that they confine permanently some of their energy in a bounded region of space) is presently the subject of a great deal of investigation. Methods to quantize classical lumps have been extensively studied (see [1–3] for example). Paradoxically, much less is known about the existence of classical lumps.

In this paper we prove that there are no classical lumps in a large class of constrained systems. In particular we prove that there are no classical lumps in the $0(N)$ non linear σ-model in 2 dimensional space-time. This example is particularly interesting because it is believed to be closely related to the Yang-Mills equations in 4 dimensional space-time, and because the $0(N)$ (in the case $N=3$) non linear σ-model in 2 dimensional space-time is believed to be equivalent to the Sine-Gordon equation [4].

From the technical point of view our result has the interesting feature that the system is not required to be scale invariant and the trace of the energy-momentum tensor is not required to be equal to zero.

There is some discrepancy in the definition of classical lumps in the literature. Here we consider a definition that contains most of the definitions found in the literature as particular cases. We will always consider non-singular solutions.

Definition I.1. A classical lump is a finite energy solution to the field equations in Minkowski space having the property that there exist ε and $R>0$ such that for

[1] They are also called solitons, classical glueballs and extended objects in the literature

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