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The Cohomology of Nets over Minkowski Space

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Abstract. We investigate the cohomology of nets over Minkowski space and develop exact sequence techniques enabling us to compute many low-dimensional cohomologies. We examine in particular nets derived from smooth solutions of invariant partial differential equations using causal support conditions. Thus the wave equation gives a trivial second cohomology whereas the vector wave equation with Lorentz condition and Maxwell's equations give a second cohomology $\mathbb{R}$ and $\mathbb{R} \times \mathbb{R}$ corresponding, respectively, to an electric and an electric and magnetic charge.

Introduction

This paper has its origins in investigations into the structure of quantum field theory. In classical (relativistic) field theory, a field can be thought of as a function $\phi(x)$ defined on space-time (Minkowski space) with values in the real line or, more ambitiously, in other manifolds. The set of fields at time $x^0 = 0$ constitute the infinite-dimensional configuration space of the system. If the fields at time $x^0 = 0$ are taken together with their conjugate momenta in the sense of Lagrangian field theory, we get the infinite-dimensional phase space of the system. Quantum fields are too singular to admit any such interpretation; they are distributions rather than functions. In any case, quantum theory does not deal directly with configuration spaces or phase spaces but instead real-valued functions on configuration space are replaced by commuting self-adjoint operators on a Hilbert space and real-valued functions on phase space by non-commuting self-adjoint operators.

A rigorous mathematical framework for quantum field theory was given by Gårding and Wightman [1, 2] who considered quantum fields to be operator-valued distributions. Here the basic objects are unbounded self-adjoint operators

$$\phi(f) = \int \phi(x) f(x) dx,$$

where f is a smooth function of compact support on Minkowski space. To relate this with the above ideas, f should be regarded as a linear function on the (linear) phase space of the system.