Commun. math. Phys. 62, 167–171 (1978)

Communications in
Mathematical
Physics

Generators of Semigroups of Completely Positive Maps

Erik Christensen

Matematisk Institut, DK-2100 Copenhagen Ø, Denmark

Abstract. Any generator of a norm continuous semigroup of completely positive normal maps on a von Neumann algebra M can be decomposed into a sum of a completely positive map and a map of the form $m \to x^*m + mx$.

The present note shows that the generator L of a uniformly continuous semigroup of completely positive normal maps on a von Neumann algebra M has the form

$$L(m) = \Psi(m) + x^*m + mx\,, \tag{$*$}$$

where Ψ is a completely positive map of M into the algebra $B(H)$ of all bounded operators on the space where M acts, and x is an operator in $B(H)$.

The problem relates to the question of whether irreversible evolutions of a quantum system come from the restriction of a reversible evolution of a larger system. Recall that ϕ is a completely positive map of a C^*-algebra $\mathscr{A}$ into $B(H)$ when ϕ is a positive linear mapping and applying ϕ to the elements of each matrix with entries in $\mathscr{A}$ yields a positive map (for matrices of all orders). By Stinespring's generalization of a result of Neumark (on positive operator-valued spectral measures), each such ϕ when identity preserving is the composition of a $*$-representation of $\mathscr{A}$ into $B(K)$ (with K a Hilbert space containing H) followed by restriction to H (i.e. $T \to PTP$ with P the projection of K onto H). If a group of $*$-automorphisms of $\mathscr{A}$ expresses a reversible dynamics, a semigroup of completely positive maps is a restriction and possibly a framework for irreversible dynamics.

The canonical decomposition $(*)$ of the generator of norm continuous semigroups of completely positive normal maps on a von Neumann algebra was first obtained independently by Gorini, Kossakowski, and Sudarshan for finite-dimensional matrix algebras [11], and by Lindblad for approximately finite-dimensional von Neumann algebras [6]. Later Lindblad [7] showed that a decomposition is possible if certain cohomology conditions are satisfied. Evans and Lewis [4] took up Lindblad's method and showed via the result of [3], that if the algebra M is properly infinite then L has a decomposition as in $(*)$.