Commun. math. Phys. 62, 159–166 (1978)

Communications in
Mathematical
Physics

Signal Propagation in Lattice Models of Quantum Many-Body Systems

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Abstract. Upper and lower bounds are proven for the speed of propagation of general classes of physical signals in particle lattice models.

1. Introduction

Several years ago Lieb and Robinson proved a remarkable result [1] consisting of an upper bound on certain commutators of operators in spin lattice models with short range interactions. Specifically, they proved for any local operators A and B, and with space (respectively time) translations denoted σ_x (respectively τ_t) that $\|[\tau_t(A), \sigma_x(B)]\|$ is $O(\exp[-K|t|])$ as $|t|\to\infty$ for some $K>0$ provided $|x|\geqq V|t|$, where V is some finite velocity, dependent on the interaction. (A cleaner proof, with simple formulas for V and K is contained in the recent paper [2].)

It was noted in [1] that this result implies an upper bound on the speed of propagation of "information" in such models, but no details were given on this interpretation.

As an indication that there are latent difficulties in this interpretation, consider the following general model of the propagation of "information" or "signals" in any quasi-local dynamical system. Let ϱ be a state, A and B local operators with "support" near the origin and such that $\varrho(A^*A)=1$. We interpret

$$S(x,t)=\varrho(A^*\tau_t\sigma_x(B)A)-\varrho(\tau_t\sigma_x(B)) \qquad (1)$$

as the signal, measured near the position x at time $t>0$ by means of the observable B, due to the disturbance localized near $x=0$ and represented by the change (effected approximately at time $t=0$) of the state $\varrho(\cdot)$ to the state $\varrho_A(\cdot)=\varrho(A^*\cdot A)$. [The condition $\varrho(A^*A)=1$ is just that ϱ_A be a state.] S measures the difference between the expected background, $\varrho(\tau_t\sigma_x(B))$, and the measured quantity, $\varrho(A^*\tau_t\sigma_x(B)A)$. To show that signals travel only with speed less than V it would be necessary to show that $S(x,t)$ is negligibly small for all $|x|>Vt$. (In this paper we do not analyze carefully the notion of "negligible"; for simplicity we just assume a

0010-3616/78/0062/0159/$01.60