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A Lower Bound for the Mass of a Random Gaussian Lattice*

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Abstract. We give a criterion that the two point function for a Gaussian lattice with random mass decay exponentially. The proof uses a random walk representation which may be of interest in other contexts.

Random mass gaussian lattices are lattice systems where the single site distribution has the form

$$\left(\int_0^\infty d\sigma(a)e^{-a\phi^2}\right)d\phi\,.$$

An example is $\frac{d\phi}{1+\phi^2}$. Related systems have been discussed quite frequently, at least in one dimension [1].[1]

Let $d\sigma(a)$ be a Borel measure on $(0,\infty)$ such that

$$\int d\sigma(a)(1+a)^{-1/2}<\infty\,. \tag{1}$$

For $\mu \geqq 0$, define

$$dm_\mu(\phi)=\left(\int d\sigma(a)e^{-(a+\mu)\phi^2}\right)d\phi\,. \tag{2}$$

Let $L_\infty \subset \mathbb{R}^d$ be a unit lattice centered on the origin, parallel to the coordinate axes. L denotes the finite part of L_∞ contained in the box $\prod_{j=1}^{d}[-l_j+1/2, l_j-1/2]$ where (l_j) are given integers. On the space $\mathbb{R}^{|L|}$, where $|L|$ denotes the number of lattice points in L, define the probability measure

$$dP_{L,\mu}=Z_{L,\mu}^{-1}\prod_{l\in L}dm_\mu(\phi_l)e^{(\phi,\Delta_D\phi)}\,, \tag{3}$$

$$(\phi,\Delta_D\phi)=-\sum_{l,l'}(\phi_l-\phi_{l'})^2\,. \tag{4}$$

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[1] The thermodynamic limit is taken after integrating over the masses, in this paper

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