Commun. math. Phys. 62, 55—70 (1978)

Communications in
Mathematical
Physics

Linear Spin-Zero Quantum Fields in External Gravitational and Scalar Fields

I. A One Particle Structure for the Stationary Case

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Abstract. We give mathematically rigorous results on the quantization of the covariant Klein Gordon field with an external stationary scalar interaction in a stationary curved space-time.

We show how, following Segal, Weinless etc., the problem reduces to finding a "one particle structure" for the corresponding classical system.

Our main result is an existence theorem for such a one-particle structure for a precisely specified class of stationary space-times. Byproducts of our approach are:

1) A discussion of when a given "equal-time" surface in a given stationary space-time is Cauchy.

2) A modification and extension of the methods of Chernoff [3] for proving the essential self-adjointness of certain partial differential operators.

§0. Introduction

In this series of papers, we discuss the quantization of the equation

$$(g^{\mu\nu}\nabla_\mu \partial_\nu + m^2 + V)\varphi = 0\,, \tag{0.1}$$

— the covariant Klein Gordon equation in a fixed curved space-time $(\mathcal{M}, g^{\mu\nu})$ and interacting with a fixed external scalar field V. (We shall always take $(\mathcal{M}, g)$, V to be C^∞.)

Notwithstanding much recent work on "quantum field theory in curved space-times" [6, 12], it is perhaps fair to say that there has been, so far, no satisfactory statement of what it would mean to quantize our Equation (0.1) on a generic space-time. For the case of stationary space-times however, there is a well established procedure — at least at the heuristic level. We ought not to rest content with this procedure while the generic case remains unsolved (most space-time are not stationary, and even for stationary space-times, why should we single out

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0010-3616/78/0062/0055/$03.20