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Phase Transitions and Reflection Positivity. I. General Theory and Long Range Lattice Models

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Abstract. We systematize the study of reflection positivity in statistical mechanical models, and thereby two techniques in the theory of phase transitions: the method of *infrared bounds* and the chessboard method of estimating contour probabilities in Peierls arguments. We illustrate the ideas by applying them to models with long range interactions in one and two dimensions. Additional applications are discussed in a second paper.

1. Introduction

Among the recent developments in the rigorous theory of phase transitions have been the introduction of two powerful techniques motivated in part by ideas from constructive quantum field theory: the method of infrared bounds [10, 4] which provides the only presently available tool for proving that phase transitions occur in situations where a continuous symmetry is broken, and the chessboard estimate method of estimating contour probabilities in a Peierls' argument [14, 9]. This is the first of three papers systematizing, extending and applying these methods. In this paper, we present the general theory and illustrate it by considering phase transitions in one and two dimensional models with long range interactions. In II, [7], we will consider a large number of applications to lattice models and in III, [8] some continuous models including Euclidean quantum field theories. Reviews of some of our ideas and those in [4, 9, 10, 14] can be found in [5, 6, 23, 27, 43]. An application can be found in [19].

Three themes are particularly emphasized in these papers. The first, §§2—4, is the presentation of a somewhat abstract framework, partly for clarification (e.g.

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