Commun. math. Phys. 61, 293—305 (1978)

Communications in Mathematical Physics
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Some Limit Theorems for Random Fields

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Abstract. We prove a central limit theorem with remainder and an iterated logarithm law for collections of mixing random variables indexed by $Z^d, d \geqq 1$. These results are applicable to certain Gibbs random fields.

1. Introduction

In this paper we extend various classical limit theorems to collections of random variables indexed by $Z^d, d \geqq 1$. The study of such collections is motivated by Gibbs random fields. For example we may define our probability space $(\Omega, \mathscr{F}, P)$ in the following way. Let $\Omega = \{-1, +1\}^{Z^d}$, with $\mathscr{F}$ the σ-algebra generated by finite-dimensional cylinder sets. Then if Φ is a translation-invariant, finite-range, real-valued potential function on the finite subsets of Z^d with $\Phi(\phi) = 0$, the Gibbs state for the potential Φ is a probability measure P on $(\Omega, \mathscr{F})$ for which, if $n \in Z^d$,

$$(1.1) \qquad P\left(\omega(n) \middle| \mathscr{F}_{Z^d - \{n\}}\right) = \left(1 + \exp\left(2 \sum_{A \ni n} (\Phi(A)) \prod_{m \in A} \omega(m)\right)\right)^{-1}$$

is a regular conditional probability distribution for the "spin" at site n given the configuration on $Z^d - \{n\}$. Here $\mathscr{F}_A$ is the σ-algebra depending on the coordinates in the set $A \subset Z^d$.

In [2] a unique such P is shown to exist for certain choices of Φ. Moreover conditions on Φ which imply that mixing occurs are stated. That is, it is shown that for certain Φ and for $A, B \subset Z^d$ the dependence of sets in $\mathscr{F}_A$ on sets in $\mathscr{F}_B$ decreases as $d(A, B)$, the usual Euclidean distance between A and B, increases, but may increase as $|A|$, the cardinality of A, increases.

In [5] the F. K. G. inequalities are used to determine conditions on Φ which imply that events at sites $n, m \in Z^d$ are positively correlated. For example if Φ is a nearest-neighbor potential

$$\Phi(\{n, m\}) > 0 \text{ for } d(n, m) = 1$$

is sufficient.

Thus we are led to study random variables $(X_n), n \in Z^d$, which are positively

* Partially supported by NSF Grant MSC 76-05828 and a grant from the Texas A&M University College of Science