Commun. math. Phys. 61, 169—175 (1978)

Communications in
Mathematical
Physics

A Uniform Lower Bound on the Renormalized Scalar Euclidean Functional Determinant

R. Schrader and R. Seiler

Institut für Theoretische Physik, Freie Universität Berlin, D-1000 Berlin 33

Abstract. Within a precise differential geometric setting we prove that the renormalized, scalar Euclidean functional determinant in an external Yang-Mills potential is bounded below by 1.

This reflects the stability of the vacuum under perturbations by external potentials. The proof is based on Kato's inequality and Seeley's analytic extension of the trace formula.

The old problem of controlling the vacuum polarization in an external field by using the functional determinant [14] has received renewed interest (see e.g. [1, 3, 5]). Within a precise differential geometric context we will prove that the renormalized scalar determinant on $\mathbb{R}^m$, given formally by

$$\Delta_{\text{ren}}(e) = \frac{\Delta(e)}{\Delta(0)} \tag{1}$$

with $\Delta(e) = \det(p - eA)^2$, satisfies the estimate

$$\Delta_{\text{ren}}(e) \geqq 1 \tag{2}$$

for all real e. A is an external Yang-Mills potential. Formally $\Delta(e)$ is the product of the eigenvalues of $(p-eA)^2$ and therefore appears in the Euclidean (functional integration) approach through the Gaussian integral over a bose field ϕ as

$$\Delta(e)^{-1} = \int \exp -1/2\phi^*(p-eA)^2\phi d\phi^* . \tag{3}$$

Relation (2) therefore states the stability of the vacuum under a perturbation by an external Yang-Mills potential [14]. In case of a spinor field in an external electromagnetic potential the corresponding stability condition

$$\frac{\det(\not p)^2}{\det(\not p - e\not A)^2} \geqq 1 \tag{4}$$

has been proved in the relativistic context by Schwinger (see Relation 112 in [14]). Presently the methods presented here do not extend to this case (for a discussion of the Spinor Laplacian in the context of Kato's inequality, see [7]).

0010-3616/78/0061/0169/$01.40