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On the Prosaic Origin of the Double Poles in the Sine-Gordon *S*-Matrix*

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Abstract. We show that the apparently exotic double poles in the sine-Gordon *S*-matrix are nothing but ordinary anomalous thresholds.

1. Introduction

The most astonishing property of the sine-Gordon *S*-matrix (other than that it is known [1]) is the host of double poles that populate the physical sheets of forward scattering amplitudes. Such double poles have long been known to be present in analogous nonrelativistic models [2]; nevertheless, it is a surprise to find them in a relativistic field theory, where we would expect all singularities to be explicable in terms of the principles of analytic *S*-matrix theory ("analyticity + unitarity") as laid down, for example, in the text of Eden et al. [3].

The purpose of this note is to explain the double poles in terms of these principles. To be more explicit, we show that the double poles are Landau singularities of a perfectly ordinary sort, just like those that occur in the real world in deuteron-deuteron scattering, for example. It is only the dimensionality of phase space that makes these singularities double poles in two dimensions and branch cuts in four dimensions. Thus the double poles have nothing to do with the special features of sine-Gordon theory (infinite number of conservation laws, factorizability, etc.) and should appear in any two-dimensional theory with appropriate mass spectrum.

In Section 2 below we explain the detailed mechanism responsible for the double poles. In Section 3 we verify our explanation by computing the locations of the double poles in the sine-Gordon *S*-matrix. We obtain all the double poles in the *S*-matrix and we obtain no double poles that do not appear in the *S*-matrix.

All our arguments are trivial and all our computations are pedestrian. That is the point of this note.

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