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Phase Transitions in Anisotropic Lattice Spin Systems

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Abstract. A general method for proving the existence of phase transitions is presented and applied to six nearest neighbor models, both classical and quantum mechanical, on the two dimensional square lattice. Included are some two dimensional Heisenberg models. All models are anisotropic in the sense that the groundstate is only finitely degenerate. Using our method which combines a Peierls argument with reflection positivity, i.e. chessboard estimates, and the principle of exponential localization we show that five of them have long range order at sufficiently low temperature. A possible exception is the quantum mechanical, anisotropic Heisenberg ferromagnet for which reflection positivity is *not* proved, but for which the rest of the proof is valid.

I. Summary of Results and General Strategy of Proofs

One of the main purposes of this paper is to explain a general method for proving the existence of phase transitions, in the sense of long range order at sufficiently low temperatures, in classical and quantum lattice systems. In principle, our method can be applied to arbitrary lattice systems satisfying *reflection positivity* (a condition closely related to the existence of a self-adjoint positive definite transfer matrix), the groundstates of which are essentially *finitely degenerate* (e.g. the space of groundstates decomposes into finitely many subspaces labelled by a discrete order parameter, sometimes related to a broken discrete symmetry group).

Our method is inspired by recent work of Glimm, Jaffe and Spencer concerning phase transitions in the $(\lambda\phi^4)_2$ quantum field model, [16]. In this paper their ideas are extended in two ways:

1. We systematize the use of *reflection positivity* and *chessboard estimates*

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