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A Generalized Lorenz System

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Abstract. A 14-dimensional generalized Lorenz system of ordinary differential equations is constructed and its bifurcation sequence is then studied numerically. Several fundamental differences are found which serve to distinguish this model from Lorenz's original one, the most unexpected of which is a family of invariant two-tori whose ultimate bifurcation leads to a strange attractor. The strange attractor seems to have many of the gross features observed in Lorenz's model and therefore is an excellent candidate for a higher dimensional analogue.

1. Introduction

In the Boussinesq approximation the equations governing atmospheric convection can be written in the following dimensionless form:

$$\begin{aligned}(\Delta\psi)_t &= -\frac{\partial(\psi,\Delta\psi)}{\partial(x,z)}+\sigma\Delta^2\psi+\sigma\frac{\partial\theta}{\partial x}\\ (\theta)_t &= -\frac{\partial(\psi,\theta)}{\partial(x,z)}+R\frac{\partial\psi}{\partial x}+\Delta\theta,\end{aligned} \tag{1}$$

where $\psi(x,z,t)$ and $\theta(x,z,t)$ denote the stream function and departure of temperature from a linear profile and σ and R denote the Prandtl and Rayleigh numbers respectively. For a derivation of the equations in the form (1) and a discussion of boundary conditions we refer the reader to [1].

In [7], Lorenz used the above system of equations as a starting point for his study on the predictability of certain atmospheric flows. By assuming that $\psi(x,z,t)$ and $\theta(x,z,t)$ had a particular form it was possible to pass from (1) to a system of three quadratically coupled ordinary differential equations, this system of equations has since become known as the Lorenz system. The Lorenz system was the

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