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Pair Function for the Rectangular Ising Ferromagnet

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Abstract. A representation of the pair correlation function for the rectangular Ising model in zero magnetic field is derived using a new spinor technique; this enables the scaling limit to be established, as well as several analytical properties of the scaling functions.

1. Introduction

In this paper a representation for the pair function will be obtained using the matrix element results of the previous paper; the equations therein to which we shall refer freely, will be denoted I.n.m. In this case the matrix element $\langle \Phi_- | G_{\alpha_n} \dots G_{\alpha_1} \sigma_1^x | \Phi_+ \rangle$ will be needed. They can be obtained in terms of the $\langle \Phi_- | G_{\alpha_n} \dots G_{\alpha_1} | \Phi_+ \rangle$ by using the definition of G_α.

By analogy with [1, Eq. (3.4)], consider the definition D.1.

$$F_M^x((e^{i\alpha})_n) = M^{n/2} \exp i \sum_1^n (\alpha_j + \Theta(\alpha_j)) \langle \Phi_- | G_{\alpha_n} \dots G_{\alpha_1} \sigma_1^x | \Phi_+ \rangle . \tag{1.1}$$

Then using [1], Eq. (2.9), (2.14), and (2.28) it follows that

$$F_M^x((z)_n) = \sum_{j=1}^n (-1)^j F_M(\Delta_j(z)_n) \left\{ 1/\Theta(z_j) + M^{-1} \sum_{z \in S_M^{\sim}} \Theta(z) f_M(z, z_j) \right\}, \tag{1.2}$$

where the functions $F_M((z)_n)$ and $f_M(z,t)$ are defined in [1, Eq. (3.4)]. The appropriate limiting form is $F^x((z)_n)$ given by

$$F^x((z)_n) = \sum_{j=1}^n (-1)^j F(\Delta_j(z)_n) \left\{ \frac{\mathscr{P}}{2\pi i} \int_{C_1} \frac{dz}{z} \Theta(z) f(z, z_j) + 1/\Theta(z_j) \right\} \tag{1.3}$$

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