Commun. math. Phys. 60, 171—180 (1978)

Communications in Mathematical Physics

Causality Criteria

Eckhard Krotscheck[1] and Wolfgang Kundt[2]

[1] I. Institut für Theoretische Physik der Universität Hamburg, D-2000 Hamburg, Federal Republic of Germany

[2] Institut für Astrophysik und Extraterrestrische Physik der Universität Bonn, D-5300 Bonn, Federal Republic of Germany

Abstract. By "causality of matter" one means its property not to admit *superluminal* excitations, i.e. excitations that propagate faster than the vacuum speed of light c. In discussing the propagation of small excitations, one has to distinguish between *phase velocities* ω_j/k, $(1 \leqq j \leqq g = \text{number of dispersion branches})$, *group velocities* $d\omega_j/dk$, a front velocity $v_f := \max_j \lim_{k\to\infty} (\omega_j/k)$, and the propagation speed $v_q := (dp/d\varrho)^{1/2}$ of isotropic quasistatic (small) perturbations. We discuss some of their properties. In particular, the (maximal) speed v_s of small signals is not smaller than v_f, and equals v_f whenever the dispersion branches $\omega_j(k)$ behave reasonably at infinity of the complex k-plane. In essence stronger conditions guarantee $v_q < v_f$ (in which case $v_q \geqq c$ would imply superluminal behaviour).

1. Introduction

Superluminal propagation velocities of perturbations have already been discussed by Sommerfeld and Brillouin [1, 2] in application to ordinary matter governed by Maxwell's equations, and have received renewed interest in the physics of matter at extreme densities, e.g. in the cores of neutron stars, cf. [3–6]. Very often in the literature one can find discussions of superluminal behaviour based on the velocity $(dp/d\varrho)^{1/2}$ which is directly formed from an equation of state $p = p(\varrho) =$ pressure as a function of mass-energy density. Though at first sight unrelated, such a procedure will receive some justification by our subsequent analysis (see in particular Proposition 5).

It is our intention to discuss and relate the fundamental velocities introduced in the abstract. To this end, our concern will be small excitations of an arbitrary extended physical system. Here the assumption "small" stands synonymously for a linearized spacetime dependence so that the solutions are superposable, and harmonic plane waves form a basis of elementary solutions. (Soliton solutions are disregarded.) Then "dispersion branches" $\omega_j = \omega_j(\boldsymbol{k})$, $(1 \leqq j \leqq g)$, govern the wave vector dependence of (angular) frequencies. This assumption does not exclude

0010-3616/78/0060/0171/$02.00