

The Onsager-Machlup Function as Lagrangian for the Most Probable Path of a Diffusion Process

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Abstract. By application of the Girsanov formula for measures induced by diffusion processes with constant diffusion coefficients it is possible to define the Onsager-Machlup function as the Lagrangian for the most probable tube around a differentiable function. The absolute continuity of a measure induced by a process with process depending diffusion w.r.t. a quasi translation invariant measure is investigated. The orthogonality of these measures w.r.t. quasi translation invariant measures is shown. It is concluded that the Onsager-Machlup function cannot be defined as a Lagrangian for processes with process depending diffusion coefficients.

1. Introduction

In the preceding years a lot of work has been done concerning the Onsager-Machlup (OM) function [1–6]. Onsager and Machlup were the first to consider the probability of paths of a diffusion process as the starting point of a theory of fluctuations [7]. Their work was restricted to processes with linear drift and constant diffusion coefficients, the generalization to nonlinear equations was undertaken by Tisza and Manning [8]. The central point was to express the transition probability of a diffusion process by means of a functional integral over paths of the process. A certain part of the integrand was then called the OM function.

Recent works are concerned with finding the correct integrand. It has often been overlooked however, that dealing with paths of a diffusion process requires an almost sure calculus. Otherwise ambiguous results may occur [3, 4, 9, 10], as was pointed out in [2, 6].

Here the OM function can only be understood as a shortened form in the functional integral mentioned above. Therefore all forms of the OM function, containing the derivative of a path, are formal expressions. This is because of the fact that almost all paths of a diffusion process are nowhere differentiable.

Still another, more physical, meaning can be given to the term OM function. Some workers have taken up the idea of Tisza and Manning to interpret the OM