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Soliton Mass and Surface Tension in the $(\lambda|\phi|^4)_2$ Quantum Field Model

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Abstract. The spectrum of the mass operator on the soliton sectors of the anisotropic $(\lambda|\phi|^4)_2$—and the $(\lambda\phi^4)_2$—quantum field models in the two phase region is analyzed. It is proven that, for small enough $\lambda > 0$, the mass gap $m_s(\lambda)$ on the soliton sector is positive, and $m_s(\lambda) = 0(\lambda^{-1})$. This involves estimating $m_s(\lambda)$ from below by a quantity $\tau(\lambda)$ analogous to the surface tension in the statistical mechanics of two dimensional, classical spin systems and then estimating $\tau(\lambda)$ by methods of Euclidean field theory. In principle, our methods apply to any two dimensional quantum field model with a spontaneously broken, internal symmetry group.

1. Introduction: Main Subject, Models, Main Results

1.1

During the past few years the quantization of nonlinear waves (solitary solutions of nonlinear,classical field equations)has attracted a lot of interest and has been studied from various—more and less rigorous—points of view; see [1–6] and references given there, and [7–10] for a mathematically rigorous analysis. From these efforts emerged the (heuristic) picture that the homotopy classes of *finite energy* solutions to some classical, nonlinear field equation are, for small enough $\hbar$ ($\propto$ Planck's constant), in a one-one correspondence with non-trivial, charged *superselection (soliton) sectors* of the relativistic quantum field theory formally determined by the same nonlinear field equation. It is felt that this picture might be a key to understanding some of the conservation laws and some of the (hadronic) extended particles observed in elementary particle physics.

So far, however, many workers in the field have concentrated on the analysis of quantum field models (or quantum spin systems [11]) in two space-time

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