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Singular Continuous Measures in Scattering Theory

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Abstract. Examples are presented of potentials V for which $-\frac{d^2}{dr^2}+V(r)$ in $L^2(0, \infty)$ has singular continuous spectrum, and the physical interpretation is discussed.

1. Introduction

Corresponding to the decomposition of a measure μ on $\mathbb{R}$ into pure point, absolutely continuous (with respect to Lebesgue measure) and singular continuous parts, in Quantum Mechanics one has a canonical decomposition of the underlying Hilbert Space $\mathscr{H}$ into the direct sum of mutually orthogonal subspaces $M_p(H)$, $M_{\text{a.c.}}(H)$ and $M_{\text{s.c.}}(H)$ [1] defined by the total Hamiltonian operator H.

In most cases, $M_p(H)$ may be taken to be the subspace spanned by the bound states of the system, and $M_{\text{a.c.}}(H)$ the subspace of scattering states (i.e. states which, in the limit as $t \to \pm\infty$, are asymptotically far from the scattering centre). For a more detailed discussion see [2, 3]. For a potential which is highly singular, and which gives rise to absorption at local singularities, $M_{\text{a.c.}}(H)$ may itself be decomposed into the subspaces respectively of scattering states and of states which are asymptotically absorbed [4].

The remaining subspace $M_{\text{s.c.}}(H)$ has usually been supposed to admit no physical interpretation (see for example [5], p. 23). Indeed, in non-relativistic potential scattering theory, considerable attention has been given to the derivation of conditions under which $M_{\text{s.c.}}(H)=\{0\}$. From the extensive literature on this subject we refer to the work of Weidmann [6] and Lavine [7, 8]. Weidmann has proved the absence of singular continuous spectrum for (non-singular) spherical potentials $V=V_1+V_2$, with V_1 of bounded variation and V_2 of short range. Lavine has proved the same result for potentials satisfying a so called "no bump" condition $\frac{r}{2}\frac{dV}{dr}+V<\text{const.}$ (These results apply in greater generality, e.g. to non-spherical potentials.) By classical analogy, this condition means that an incoming particle will encounter no effective obstacle and will ultimately recede to infinity having been scattered by the potential.

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