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Some Remarks on the Gribov Ambiguity

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Abstract. The set of all connections of a principal bundle over the 4-sphere with compact nonabelian Lie group under the action of the group of gauge transformations is studied. It is shown that no continuous choice of exactly one connection on each orbit can be made. Thus the Gribov ambiguity for the Coloumb gauge will occur in all other gauges. No gauge fixing is possible.

§ 1. Introduction and Notation

Let $\mathfrak{A}$ denote the set of all vector potentials (connections) for a fixed principal bundle P with gauge group G over a base space M. The group of the bundle G is a compact nonabelian group and M is an oriented Riemannian manifold. For each $A \in \mathfrak{A}$, let F_A denote the field (curvature two form with values in the Lie algebra g of G) of A and $\|F_A\|^2$ the norm square or action. Let $\mathfrak{G}$ denote the group of gauge transformations of P (the automorphisms of P which induce the identity map on M). Since $\mathfrak{G}$ is a group of transformations on P, it induces a group of transformations on $\mathfrak{A}$. If $\varphi \in \mathfrak{G}$ and $A \in \mathfrak{A}$, we denote this transformation by $\varphi \cdot A$. It is easy to see that $\varphi \cdot A = A - (D_A \varphi)\varphi^{-1}$ where D_A is the covariant differential. In local coordinates, $D_A = \frac{\partial}{\partial x_\gamma} + A_\gamma$ so that $(\varphi \cdot A)_\gamma = -\frac{\partial \varphi}{\partial x_\gamma}\varphi^{-1} + \varphi A_\gamma \varphi^{-1}$. Also, $F_{\varphi \cdot A} = \varphi F_A \varphi^{-1}$ so that $\|F_{\varphi \cdot A}\| = \|F_A\|$.

Let $\mathfrak{N} = \mathfrak{A}/\mathfrak{G}$ the orbit space of $\mathfrak{A}$ under $\mathfrak{G}$, i.e., the set of equivalence classes where A and B are equivalent if there exists a $\varphi \in \mathfrak{G}$ such that $B = \varphi \cdot A$. In the Feynman approach to quantum field theory one wants to make sense out of $\int_{\mathfrak{A}} e^{-\|F_A\|^2}\{\ \}\mathscr{D}A / \int_{\mathfrak{A}} e^{-\|F_A\|^2}\mathscr{D}A$, where the integrand of the numerator may be constant on orbits of $\mathfrak{G}$. This introduces a difficulty because the orbits are expected to have infinite measure. One should really integrate over $\mathfrak{N}$, an intractable space. The physicists attempt to get round this difficulty by choosing a particular gauge, that is, choosing in a continuous manner one vector potential on each orbit; this choice is a map $s: \mathfrak{N} \to \mathfrak{A}$ such that $p \circ s = I$ where $p: \mathfrak{A} \to \mathfrak{N}$ is projection. They

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