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Correlation Inequalities and the Thermodynamic Limit for Classical and Quantum Continuous Systems*

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Abstract. We use Ginibre's general formulation of Griffiths' inequalities to derive new correlation inequalities for two-component classical and quantum mechanical systems of distinguishable particles interacting via two body potentials of positive type. As a consequence we obtain existence of the thermodynamic limit of the thermodynamic and correlation functions in the grand canonical ensemble at arbitrary temperatures and chemical potentials. For a large class of systems we show that the limiting correlation functions are clustering. (In a subsequent article these results are extended to the correlation functions of two-component quantum mechanical gases with Bose-Einstein statistics). Finally, a general construction of the thermodynamic limit of the pressure for gases which are not H-stable, above collapse temperature, is presented.

1. Systems of Particles Interacting via Two Body Potentials of Positive Type

In this paper we study classical and quantum continuous systems in thermal equilibrium. These systems consist of particles the interactions among which are described by two body potentials of positive type. We are interested in proving the existence of the thermodynamic limit of the pressure and the Gibbs equilibrium states in the grand canonical ensemble. We also want to discuss certain properties of the equilibrium states in the thermodynamic limit, *such as clustering.* Two classes of systems are considered:

(C) Classical particles, and

(QM) Quantum mechanical, distinguishable particles, ("Boltzmann statistics").

The particles have internal degrees of freedom which we call (generalized) charges. They are labelled by the vectors q of some measurable vector space Q.

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