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The Particle Structure of ν-Dimensional Ising Models at Low Temperatures

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Abstract. In this work we study the ν-dimensional Ising model at low temperatures and establish the existence of an upper gap in the energy-momentum spectrum of the two-point function for $\nu \geqq 3$. For $\nu = 2$, it is known that this gap is absent.

1. Introduction

The low energy structure of the energy-momentum spectrum of a Quantum Field Theory is to a large extent determined by the asymptotic behavior of pair correlation functions. It is also connected with the particle structure of the theory. Results in this direction have been obtained for some nontrivial models in two and three space-time dimensions [7, 15, 1]. While four dimensional models have not yet been constructed, we can study other physical systems which are simple enough to be realized in any dimension and yet have some resemblance to a field theory. One example is the ν-dimensional Ising model, to be compared with the $(\nu-1)$ (space) $+1$ (imaginary time) Euclidean field theory. In this work, we study the particle structure of that model and obtain results that indicate the existence of isolated one particle states when $\nu > 2$ for sufficiently low temperatures. Aside from its connection with field theory, the problem is interesting in itself because of the remarkable difference between the cases $\nu = 2$ and $\nu > 2$. This is manifest in the asymptotic behavior of the pair correlation function, which for $\nu = 2$ violates its expected decay rate at infinity (Orstein-Zernike prediction). The Orstein-Zernike prediction is a consequence of the particle structure we derive here together with further properties of the dispersion relations for one particle states [11], which in principle could be studied by the methods developed in this work. The failure of the Orstein-Zernike behavior for $\nu = 2$ at low temperatures is due to a failure of the required particle structure and is related to the existence of solitons for $\nu = 2$ only, [14]. In Section III, we will give a simple geometric picture to explain the

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