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# An Inequality for Fermion-Systems

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**Abstract.** It is shown that for certain classes of Euclidean fermion-boson systems on a lattice vacuum expectation values of scalar fields increase if a Yukawa-interaction is turned on. Applicability and possible extensions of this result in the framework of constructive quantum-field-theory are discussed.

## 1. Introduction

In recent times much of the progress made in constructive field theory has been due to the extensive use of correlation inequalities such as the Griffiths and Lebowitz inequalities ([1, 2] where original work is quoted). However, until now, the range of applicability of this powerful tool appears to have been limited to purely scalar theories. In this paper I want to show that by applying Griffiths' inequality one can derive an inequality for systems containing not only a scalar or[1] a pseudoscalar field but also a Majorana- or[1] Dirac-spinor.

To state the main result some notation must be introduced that will be used in the sequel. As the UV-limit will not be considered in this article the models are formulated on a periodic Euclidean space-time lattice $\mathscr{T}$[2] with lattice-spacing $a$ whose elements will be denoted by $k, m, n, \ldots$ where $n=(n_0, n_1, n_2, n_3)$; because of periodic boundary conditions there exists $N \in \mathbb{N}$ such that $n_\mu$ and $n_\mu + N$ are to be identified. Then the type of theory I will consider contains the following parts:

1) Bosonic part of the action

$$S_0(A) = a^4 \sum_{n \in \mathscr{T}} \left\{ \frac{Z_0}{2} (\nabla_\mu A_n)^2 + \frac{m_0^2}{2} A_n^2 + \lambda A_n^4 \right\}$$
$$S_0(B) = a^4 \sum_{n \in \mathscr{T}} \left\{ \frac{Z_0}{2} (\nabla_\mu B_n)^2 + \frac{m_0^2}{2} B_n^2 + \lambda B_n^4 \right\}^3 , \tag{1.1}$$

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1 Exclusive "or"

2 For simplicity, I restrict myself to the case $d=4$. Most of the results of this paper can be recovered for $d=2$ and $d=3$ in a straightforward manner

3 $\nabla_\mu \varphi_n := \frac{1}{2a} (\varphi_{n+\hat{\mu}} - \varphi_{n-\hat{\mu}})$