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Analyticity and Clustering Properties of Unbounded Spin Systems*

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Abstract. Transforming any lattice system in a polymer model, we use known analytic and cluster properties of the latter to derive similar ones for general lattice models with two-body interactions. These properties of the lattice model hold when the temperature is high enough.

Introduction

Our purpose here is to study various lattice models in some weak coupling regime. In particular we will prove that, at high enough temperatures, the free energy and the correlation functions of these models are analytic functions of any parameter on which the hamiltonian depends analytically. Moreover, under the same conditions and for finite range interactions, the two-point functions will be proved to decay exponentially.

These models contain as special cases, lattice gases with two-body interactions, classical Heisenberg models and lattice approximations of field theoretical models. They describe also some anharmonic crystals, of interest for ferroelectricity. Results of this kind were already obtained for some of these models [1, 2]. However, the technique used relied heavily on further properties of the model in question, such as the boundedness of the values taken by the spin variables. They could not therefore be generalized to lattice approximations of field models for example.

Our strategy here is the following: we transform any lattice model in a so-called polymer model, which can be seen as a generalized lattice gas, with hard core interactions. These polymer models were studied previously [3, 4] and various analytic and clustering properties were established for their gaseous phase (i.e. in the weak coupling region). The remaining task is therefore to estimate the parameters of the polymer model in terms of those of the corresponding lattice models. This is done in the case of two-body interactions only in order to simplify as much as possible the analysis. Our expansion is very much related to the old

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