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Odd Operators and Spinor Algebras in Lattice Statistics: n-Point Functions for the Rectangular Ising Model*

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Abstract. An extension is given of the development by Schultz et al. [3] of the pioneering work of Kaufman and Onsager [2] on the planar Ising ferromagnet. This involves a novel form of Wick's theorem [19] for fermions. In subsequent papers the method will be applied to determine the n-point functions and to evaluate rigorously critical indices.

I. Introduction

The algebraic method of computing quantities in the Ising model requires the spectral decomposition of the transfer matrix and the evaluation, in the basis generated thereby, of matrix elements of appropriate operators. The purpose of this paper and the following one is to give a complete determination of all such matrix elements for the planar ferromagnet with nearest neighbour interactions and subject to cyclic boundary conditions. The techniques pioneered by Onsager [1] and Kaufman [2], together with their subsequent development by Schultz et al. [3], will be extended. Thus the present work is quite unlike that of Wu and coworkers [4] in which combinatorial concepts and Pfaffians are introduced at the outset. Further, the results given here are rigorous. The principal application reported here is the determination of n-point functions, [5], also obtained by McCoy et al. [6]. The 4-point function has been investigated in some detail by Au Yang [7]. Previous applications of these results are the proof that the susceptibility divergence exponents are $\gamma=\gamma'=7/4$ [8] and the investigation of the interface between coexisting pure phases [9]. To put the results in context, the Ising model will now be defined: let $\mathbb{Z}^d$ generate the d-dimensional hypercubical crystal lattice with unit edge, the Cartesian coordinates of each lattice site, called a vertex, being $\boldsymbol{r}=(r_1 \dots r_d)$, $r_i \in \mathbb{Z}$. At each vertex $\boldsymbol{r}$ there is a spin $\sigma(\boldsymbol{r})=\pm 1$. A spin configuration $\{\sigma\}$ on a sub-box Λ is defined by specifying $\sigma(\boldsymbol{r})$ for each $\boldsymbol{r} \in \Lambda$, and

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