

A Rigorous Approach to Debye Screening in Dilute Classical Coulomb Systems

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Abstract. The existence and exponential clustering of correlation functions for a dilute classical coulomb system are proven using methods from constructive quantum field theory, the sine gordon transformation and the Glimm, Jaffe, Spencer expansion about mean field theory. This is a vindication of a belief, of long standing amongst physicists, known as Debye screening. This states that, because of special properties of the coulomb potential, the configurations of significant probability are those in which the long range parts of r^{-1} are mostly cancelled, leaving an “effective” exponentially decaying potential acting between charge clouds.

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