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Towards a Constructive Approach of a Gauge Invariant, Massive $P(\phi)_2$ Theory

Robert Schrader

Institut für theoretische Physik, Freie Universität Berlin,
D-1000 Berlin 33, Federal Republic of Germany

Abstract. As part of a possible constructive approach to a gauge invariant $P(\phi)_2$ theory, we consider massive, scalar, polynomially selfcoupled fields ϕ in a fixed external Yang-Mills potential A in two-dimensional euclidean space. For a large class of A's we show that the corresponding euclidean Green's functions for the fields ϕ have a lower mass gap for weak coupling which is uniform in A. The result is obtained by adapting the Glimm-Jaffe-Spencer cluster expansion to the present situation through Kato's inequality, which reflects the diamagnetic effect of the Yang-Mills potential. A discussion of the corresponding gauge covariance is included.

1. Motivation and Outline of Results

There is an increasing belief that Yang-Mills field theories should play an important role in the description of elementary particles. Now the recent attempts to get a rigorous mathematical grip on the problems related to Yang-Mills fields mostly start with a lattice formulation (see e.g. the contributions to the Rome conference on Mathematical Physics and the references quoted there). However, there is *one* aspect which directly allows a continuum discussion and which is the object of the present analysis. For definiteness, we consider a gauge invariant $P(\phi)_2$ theory, but we expect that our arguments may be extended to gauge invariant Y_2 (Yukawa) and ϕ_3^4 theories.

In our case the euclidean Green's functions should formally be given as the moments of a normalized measure μ_f with

$$d\mu_f(\phi^*,\phi,A)=Z_f^{-1}\prod_x \delta(f(A(x)))\exp-\int(\tfrac{1}{4}(F^{\mu\nu}F_{\mu\nu}+P(\phi^*,\phi))(x)dx \cdot\exp-\int(\phi^*(-\Delta_A+m_0^2)\phi)(x)dx\prod_{x,i,\mu} d\phi_i^*(x)d\phi_i(x)dA_\mu(x) \tag{1.1}$$

with

$$\Delta_A=\sum_\mu(\partial_\mu+ieA_\mu)^2 \tag{1.2}$$

(see e.g. Faddeev and Popov [4, 13], Abers and Lee [1]).

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