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# The Singular Holonomy Group

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**Abstract.** The "fibre" of the extension of the frame-bundle of a space-time over a $b$-boundary point $p$ is a homogeneous space $\mathscr{L}/G_p$. It is shown that $G_p$ can be found by a construction like that for a holonomy group, and that it contains a subgroup determined by the Riemann tensor. Near a curvature singularity one would expect $G_p = \mathscr{L}$.

## 1. Introduction

A singular space-time is one in which there is a curve $\gamma$ (not necessarily causal) that cannot be extended further in the direction of increasing parameter—i.e. it does not stop at a point in space-time—and that has finite (Euclidean) length measured in a frame parallely propagated along it [6]. At the time when Hawking, Penrose and Geroch showed that being singular could be a general property of space-time, attempts were made to define singular points, endpoints of such incomplete curves, that formed a boundary to space-time. In particular, Schmidt [6] produced the elegant construction of the $b$-boundary involving a natural Cauchy completion of the bundle of all (pseudo-)orthonormal frames (i.e. frames with respect to the Lorentz metric). The usefulness of this as a means of providing a canonical boundary has been disputed; but it certainly can provide valuable insight into what is going wrong at a singularity, into why a curve is forced to be incomplete.

We shall see that a particular group arising in the course of Schmidt's construction contains information about the unbounded part of the curvature, and about the "topological" peculiarities that can complicate a singular situation. We thus have a tool for separating and classifying different aspects of singularities, a necessary step towards understanding their physical significance.

In forming the $b$-boundary one first forms the closure $\mathrm{Cl}_b L(M)$ of the (pseudo-)orthonormal frame bundle $L(M)$ with respect to a positive definite

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