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Passive States and KMS States for General Quantum Systems

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Abstract. We characterize equilibrium states of quantum systems by a condition of passivity suggested by the second principle of thermodynamics. Ground states and β-KMS states for all inverse temperatures $\beta \geqq 0$ are completely passive. We prove that these states are the only completely passive ones. For the special case of states describing pure phases, assuming the passivity we reproduce the results of Haag et al.

Introduction

The main aim of the equilibrium statistical physics is a description and investigation of equilibrium states for large physical systems. To this end we use infinite systems as good mathematical models.

The physical motivation for this paper is a question; how to describe the equilibrium states for a given infinite quantum system?

As it is well known, in the traditional approach we consider the finite systems for which the equilibrium states are better known and then we take the thermodynamical limit. If the limit exists, one assumes that it describes the equilibrium state of the infinite system.

Starting with the Gibbs canonical ensemble characterized by inverse temperature β for finite systems and keeping β constant, one can easily prove that the limit state is β-KMS state for the evolution group of the infinite system. In other words, if H is a generator of this group then the modular automorphism group associated with this state is given by $-\beta H$.

The KMS states are formal generalizations of the canonical Gibbs states for infinite systems and it is not obvious whether they possess properties attributing to equilibrium states.

Some number of papers are devoted to answer this question. For quantum lattice systems Araki proved that every KMS state is a limit of Gibbs states [1],