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Occurrence of Whimper Singularities

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Abstract. Homogeneous space-times (i.e. those admitting a three-parameter group of isometries) are studied using the Newman Penrose formalism. It is found that solutions containing horizons depend on two fewer parameters than the most general solution, so that horizons and the associated whimper singularities are not stable features of homogeneous space-times. In the vacuum case, there are just three two-parameter families with horizons, two of which are the NUT solutions and certain plane waves.

1. Introduction

Singularities in space-time are characterised by the existence of an incomplete curve, $\gamma(v)$, with $0 \ll v < v_+$ (see [1–3]). In particular, the more physical types of singularity occur if components of the Riemann tensor fail to tend to finite limits when measured in a frame which is parallelly propagated along $\gamma(v)$; these are called curvature (or p.p.)[1] singularities. In this case the space-time cannot be extended through the singular point in any reasonable way. Curvature singularities can be subclassified as either c^0 scalar (curvature)[1] or c^0 non-scalar (intermediate). If v_+ is a c^0 non-scalar singularity, all polynomial invariants of the Riemann tensor tend to finite limits as $v \to v_+$; or equivalently [4], there exists a (non-parallelly propagated) frame in which components of the Riemann tensor do tend to finite limits. If such a frame does not exist, or if some polynomial invariant of R_{abcd} is badly behaved, the singularity is c^0 scalar.

Although it is known [5] that general, physically reasonable space-times must become singular somewhere, very little is known about the type of singularity which occurs. One approach to this problem is to investigate the behaviour of all members of a restricted class of solutions: if then the less desirable types of singularity turned out to arise only in a small subclass of these solutions, one could more readily believe that they are not a general feature of realistic universe models.

[1] The classification here is due to Ellis and Schmidt [1] and differs from that given by Ellis and King in [2]. To avoid confusion, the terminology of [1] will usually be accompanied in parentheses by the equivalent description from [2]