Commun. math. Phys. 58, 241—254 (1978)

Communications in Mathematical Physics

Generating Functions for the Affine Symplectic Group

G. Burdet* **, M. Perrin**, and M. Perroud***

Centre de Recherches Mathématiques, Université de Montréal, Montréal H3C3J7, Canada

Abstract. The generating function notion is used to give a representation of the inhomogeneous symplectic group as group of affine canonical transformations. Then the classical action for linear mechanical systems, the Hamiltonians of which belong to the algebra $h\,\mathrm{sp}(2n, \mathbb{R})$, is deduced; it is explicitely constructed for all the Hamiltonians belonging to some particular subalgebras of $h\,\mathrm{sp}(2n, \mathbb{R})$. The metaplectic representation of $W\,\mathrm{Sp}(2n, \mathbb{R})$ on $L^2(\mathbb{R}^n)$ and the solutions of the Schrödinger equation for linear systems are also obtained in terms of generating functions. The Maslov index is explicitly constructed for the quantum corresponding sets of Hamiltonians considered in the classical case.

Introduction

It is well known that the symplectic group $\mathrm{Sp}(2n, \mathbb{R})$ is connected with $\mathbb{Z}$ as fundamental group. Hence it possesses a non trivial two-sheeted covering $\mathrm{Sp}_2(2n, \mathbb{R})$ a unitary representation of which (now called the metaplectic representation) has been studied in [1] in quantum mechanical and field theory contexts. An explicit realization in terms of integral operators on $L^2(\mathbb{R}^n)$ was given in [2], which makes appear the so-called Maslov index [3].

This representation is contained in a projective unitary representation of a larger group the inhomogeneous symplectic group, which is a true unitary representation of the group $W\,\mathrm{Sp}(2n, \mathbb{R})$ obtained by extending $\mathrm{Sp}_2(2n, \mathbb{R})$ by the Weyl group.

A large set of quantum mechanical problems is then solved, namely the one's the evolution operator of which is a one-parameter subgroup of $W\,\mathrm{Sp}(2n, \mathbb{R})$ (i.e. linear quantum mechanical systems).

---

* Département de Mathématiques, Université de Montréal, Montréal H3C 3J7, Canada

** Members of the Centre National de la Recherche Scientifique (France)

Present address: Laboratoire de Physique Mathématiques, Faculté des Sciences Mirande, F-21000 Dijon, France

*** Recipient of aid from the Ministère de l'Education du Gouvernement du Québec

0010-3616/78/0058/0241/$02.80