Commun. math. Phys. 58, 223—240 (1978)

Communications in
Mathematical
Physics

The Construction of Self-dual Solutions to SU(2) Gauge Theory

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Abstract. Ignoring the problem of sources and singularities, explicit expressions are constructed for the ansätze of Atiyah and Ward. These take an especially simple form in the R gauge of Yang. Some non-linear transformation properties of the self-duality equations in this gauge provide an inductive proof of the ansätze. There is a six-parameter family of these Bäcklund transformations. They take real SU(2) gauge fields into real SU(1, 1) gauge fields and vice versa.

1. Introduction

During the past year a great deal of progress has been made towards an understanding of classical gauge field theory, particularly for an SU(2) gauge group in a four dimensional Euclidean space. If it is assumed that the field strengths are self-dual (or anti-self-dual) the analysis can be taken much further, though this assumption has, as yet, no particular physical motivation. Indeed Ward [1] has shown how all the information contained in a self-dual gauge field can be "coded" into the structure of certain analytic complex vector bundles, the isomorphism class of the bundle being determined by the gauge fields. More importantly, the isomorphism class of the bundle determines the gauge fields up to a gauge transformation and Ward showed how, in principle, the fields may be extracted from the bundles.

Taking this approach further, Atiyah and Ward [2] used basic theorems in geometry to argue that these bundles are necessarily algebraic and to restrict further the bundle structures to be considered to obtain all self-dual SU(2) gauge fields. Thus the problem of finding the $(8k-3)$-parameter family [3] of solutions was reduced to one in algebraic geometry. (Here k denotes the instanton number.) Their construction leads to a hierarchy of ansätze, $A_l (l=1,2,\ldots)$, the l-th one of which can be expressed in a form which has as input the components of a "spin $(l-1)$" massless anti-self-dual linear field. The first ansatz, A_1, had been known for

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0010-3616/78/0058/0223/$03.60