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On the Characterization of Relativistic Quantum Field Theories in Terms of Finitely Many Vacuum Expectation Values. I

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Abstract. A characterization of monotone continuous linear functionals on tensoralgebras which arise in QFT is derived and some consequences are investigated. Then we look for necessary and sufficient conditions on a set

$$T_{(N)}=\{1, T_1, T_2, \dots, T_N\} \qquad T_n \in E'_n$$

of "n-point-functionals", which guarantee the existence of at least one monotone continuous linear functional

$$S=\{1,S_1,S_2,\dots\} \quad \text{on} \quad \underline{E}=\bigoplus_{n=0}^{\infty} E_n, \qquad E_n=E_1 \tilde{\otimes}_\pi E_1 \tilde{\otimes}_\pi \dots \tilde{\otimes}_\pi E_1,$$

E_1 a special nuclear space, such that $S \upharpoonright \bigoplus_{n=0}^{N} E_n = T_{(N)}$, with special attention to QFT. A first application is a characterization of all monotone continuous linear extensions in the case $N=2$. The notion of minimal extensions is introduced. Its relevance is discussed. Necessary and sufficient conditions on $T_{(2N)}$ for the existence of minimal extensions are presented. Some properties of minimal extensions are derived. In the simplest case $E \cong \mathbb{C}$ the concept of minimal extensions allows to answer the extension problem completely for arbitrary $N \in \mathbb{N}$. For the case of general $E=E_1$ and $N=2$ it is shown that the known examples of monotone continuous linear extensions are minimal extensions or a special generalization of it.

0. Introduction, Notation

Up to now the main concern in axiomatic QFT has been that of the linear program [1,4,8]. The nonlinear constraints of QFT (positivity condition and uniqueness of the vacuum) have been treated much less [15, 17, 19]. But nevertheless these nonlinear constraints are as important as the linear constraints are. One step of incorporating the positivity condition into QFT is the program of Bros and Lasalle [20]. It relies on additional technical assumptions.

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