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Selfadjointness of the Liouville Operator for Infinite Classical Systems*

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Abstract. We study some properties of the time evolution of an infinite one dimensional hard cores system with singular two body interaction. We show that the Liouville operator is essentially antiselfadjoint on the algebra of local observables. Some consequences of this result are also discussed.

1. Introduction

In the last years the time evolution of infinite classical particle systems has been studied by various authors [1–8]. The main problem to solve was to give an existence theorem for the infinite equations of motion which formally read as:

$$\left.\begin{aligned} &\frac{d}{dt}p_i(t)=F_i(q(t))\\ &\frac{d}{dt}q_i(t)=p_i(t)\\ &p_i(0)=p_i \qquad q_i(0)=q_i, \qquad i\in\mathbb{Z} \end{aligned}\right\} \tag{1.1}$$

where $q(t)\equiv\{q_i(t)\}_{i=-\infty}^{i=+\infty}$ and $p(t)\equiv\{p_i(t)\}_{i=-\infty}^{i=+\infty}$ are respectively the positions and momenta of the particles and $F_i(q(t))$ is the force on the "i-th" particle induced by the others, and p_i, q_i are the initial data. A trajectory $(p,q)\to(p(t),q(t))$ satisfying Equations (1.1) for all $t\in\mathbb{R}$ may be found if we make suitable hypothesis on the interactions and on the regularity of the initial conditions (p,q). The set $\hat{\mathfrak{X}}$ of these couples (p,q) (phase points) is large enough to have full measure with respect to the equilibrium measure ν, and this allows to construct a triple $(\mathfrak{X}, S_t, \nu)$ where $\mathfrak{X}$ is the phase space and S_t is a ν-almost everywhere defined one parameter group of ν-invariant transformations satisfying Equations (1.1). Nevertheless till now very little is known about the physical properties of the dynamics; for example it is possible to exhibit explicitely the initial conditions $x\in\hat{\mathfrak{X}}$ for which S_tX is defined only in the one dimensional case [1, 7] and, by the choice of particular interactions,

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