Commun. math. Phys. 58, 85—112 (1978)

Communications in Mathematical Physics

On Resonant Classical Hamiltonians with Two Equal Frequencies

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Abstract. This paper contains a detailed study of the flow that the classical Hamiltonian

$$H=\tfrac{1}{2}(x_1^2+y_1^2)-\tfrac{1}{2}(x_2^2+y_2^2)+\mathcal{O}_3$$

induces near the origin of its phase space R^4. Here the perturbation term $\mathcal{O}_3$ represents a convergent power series. In particular, criteria for the existence and stability of periodic orbits are developed and expressed in terms of canonical invariants that are extracted from the perturbation term.

0. Introduction

The present work is dedicated to a study of the flow that the Hamiltonian (1.1) induces near the origin of its phase space R^4.

The methods that we will apply in our study are essentially the same as those of [1]. In that paper a Hamiltonian was studied which can be obtained from the Hamiltonian (1.1) of the present work by replacing the minus sign in the leading term

$$L=\tfrac{1}{2}(x_1^2+y_1^2)-\tfrac{1}{2}(x_2^2+y_2^2)$$

by a plus sign. The symplectic transformations that leave the leading term L invariant constitute the group $U(1,1)$, and correspondingly the Gustavson normal form of our Hamiltonian is best viewed as a function of a canonical set: $M_0=\frac{L}{2}$, M_1, M_2, M_3 of generators of that group, that is to say, as a function over the Lie-algebra $u(1,1)$. Here L or M_0 generates the center of the group, whereas M_1, M_2, M_3 generate SU(1, 1).

We bring the Hamiltonian (1.1) into normal form up to order $2n$, where n is determined by the condition that the lowest degree, non-trivial polynomial K_n in the generators of the group $U(1,1)$ that appears in the normal form is homogeneous of degree n. Here, a polynomial is called non-trivial if it is not just a function of $M_0=\frac{L}{2}$

0010-3616/78/0058/0085/$05.60