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The Boltzmann Equation

I. Uniqueness and Local Existence*

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Abstract. An abstract form of the spatially non-homogeneous Boltzmann equation is derived which includes the usual, more concrete form for any kind of potential, hard or soft, with finite cutoff. It is assumed that the corresponding "gas" is confined to a bounded domain by some sort of reflection law. The problem then considered is the corresponding initial-boundary value problem, locally in time.

Two proofs of existence are given. Both are constructive, and the first, at least, provides two sequences, one converging to the solution from above, the other from below, thus producing, at the same time as existence, approximations to the solution and error bounds for the approximation.

The solution is found within a space of functions bounded by a multiple of a Maxwellian, and, in this space, uniqueness is also proved.

1. Introduction

This is the first in a projected sequence of papers on solutions of the Boltzmann equation in a domain V. Here, we restrict our attention to the local matters of uniqueness and local existence, needed in our later papers. In these subsequent papers, we hope to address questions of global existence, approximate solutions, numerical computation of solutions, and other matters.

The existence theorem we prove here is of interest for a number of reasons. First, with a single exception, it is the only such result we know of that applies to the spatially inhomogeneous Boltzmann equation. The exception is the theorem of Grad [1, §20] which, first of all, is limited to what we call *soft* interactions, and, more important, does not treat a physical domain with a boundary. Our result, on the other hand, applies to a non-homogeneous gas that is confined to a domain V by means of a reflection law and in which either soft or hard interactions may take place.

Second, we find the solution by strictly constructive means. We show that it is a limit of a sequence of functions that are themselves solutions of easily solved, first

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