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On the Existence of Crossing Symmetric and Unitary Scattering Amplitudes with Regge Poles

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Abstract. We give a complete proof of the existence of scattering amplitudes $A(s, t, u)$ with the following properties

i) the amplitudes are total symmetric in s, t, and u,

ii) they satisfy elastic unitarity for $4 \leqq s \leqq 16$, and

iii) they develop resonances for $l \geqq 2$ on a bounded Regge trajectory which dominates the asymptotics for large energies.

I. Introduction

A rather general class of $\pi\pi$ scattering amplitudes which satisfy exactly crossing symmetry elastic unitarity and unitarity bounds in the inelastic region is known to exist [1]. But among the solutions of these papers there are no amplitudes with a Regge pole asymptotics in the physical region. More generally the methods of [1] do not allow to derive amplitudes which by construction show at least one of the following properties

i) for fixed t the asymptotics in s is exactly powerlike, e.g. $\operatorname{Im} A(s, 0) \simeq \text{const} \cdot s$ for $|s| \to \infty$,

ii) the asymptotics in s depends on the value of t in the region $t < 16$.

These restrictions are caused by the technique to work with the Mandelstam integral [2]. To obtain amplitudes which are dominated by Regge poles and satisfy crossing symmetry and elastic unitarity one has to evaluate elastic unitarity with an other method.

There are essentially two ways to proceed, either to use the Watson Sommerfeld representation with complex angular momenta, or to work with the Mellin transformation, i.e. the Khuri representation of the scattering amplitude [3]. In the frame work of complex angular momenta elastic unitarity becomes a trivial equation, the difficulties arise from analyticity and crossing symmetry. In a series of publications Atkinson, Warnock and their collaborators are investigating this method [4, 5].