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Causal Logic of Minkowski Space

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Abstract. It is shown that double-orthogonal sets (diamonds) in Minkowski space form an orthomodular complete lattice. Connection with empirical logic of Randall and Foulis is discussed.

1. Introduction

Relations of a causal type have been studied by various authors (see e.g. [1], and references there). In particular, Zeeman [2] has shown that every automorphism of the causal order structure of the Minkowski space is either Poincaré transformation or dilation. This result has been generalized in [3–5], and finally Borchers and Hegerfeldt [6] have been able to show that preservation of light cones is sufficient to deduce the Zeeman's result (plus time inversion). On the other hand, the present authors [7], motivated by quantum mechanics and algebraic quantum field theory, started investigations of a causal-logic struture of space-time. In the Galilean case the most general covariant representation of the causal logic has been found to correspond to the quantum mechanics of an extended body. This was possible owing to the extremely simple structure of the Galilean logic, which happens to be nothing but a disjoint sum of Boolean logics. The causal structure of the Minkowski space is much less trivial. The present paper shows that the family of all double-orthogonal sets is a complete orthomodular lattice (in fact, not a modular one). The full group of automorphisms of this logic consists of dilations and Poincaré transformations. In relativistic quantum mechanics one should have a covariant representation of this logic in the logic of projections of a Hilbert space. In quantum field theory projections are to be replaced by more general non-Abelian von Neumann algebras.

The methods and language we have found most appropriate are those of empirical logic of Randall and Foulis [8]. On the other hand, our example illustrates neatly all of the main concepts of Randall and Foulis approach.