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The Existence of Non-trivial Asymptotically Flat Initial Data for Vacuum Spacetimes

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Abstract. This paper demonstrates the existence of non-trivial solutions (g, k) to the constraint equations of the initial value formulation of the Einstein field equations over $\mathbb{R}^3$ with $g_{ij}-\delta_{ij}\sim|x|^{-1}$ as $|x|\to\infty$. Using the conformal methods of Lichnerowicz and York, this problem is divided into two parts. First, using weighted Sobolev spaces it is shown the set of pairs (g, k) with g a conformal metric and k transverse-traceless with respect to g forms a smooth vector bundle $\mathscr{P}$ with infinite dimensional fiber. Second, it is shown that the elements of a large open set in $\mathscr{P}$ uniquely determine a solution to the scalar constraint equation with the appropriate growth at infinity, and thereby determine solution to the constraint equations.

1. Introduction

In writing the Einstein field equations for a vacuum space-time as an evolution system on a 3-manifold M, one finds the Cauchy data consists of a Riemannian metric g_{ij} and a symmetric covariant 2-tensor π_{ab} satisfying the constraint equations (see Marsden [15]):

$$\begin{cases}\operatorname{div}_g \pi = 0 \\ \pi\cdot\pi - \frac{1}{2}(\operatorname{tr}_g \pi) - R(g) = 0 .\end{cases} \tag{1}$$

Our notation using the summation notation is:

$$\operatorname{div}_g \pi = \pi^{ab}_{\ \ |b}\,, \quad \pi\cdot\pi = \pi^{ab}\pi_{ab}\,, \quad \operatorname{tr}_g \pi = g^{ab}\pi_{ab}$$

and $R(g)$ is the scalar curvature of g. Of course covariant differentiation is done with respect to g. If M is thought of as a spacelike hypersurface embedded in a spacetime V^4, and g_{ab} and k_{ab} is the induced metric and second fundamental form of the embedding, then $\pi_{ab}=((\operatorname{tr}_g k)g_{ab}-k_{ab})$. (Note, we will use tensors and not densities.)

The constraint equations form a coupled non-linear system of partial differential equations. The existence of solutions to this system has received much attention

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