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The Non-relativistic Limit of $\mathscr{P}(\varphi)_2$ Quantum Field Theories: Two-Particle Phenomena

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Abstract. It is proved that for two-particle phenomena the $\mathscr{P}(\varphi)_2$ quantum field theories with speed of light c converge to non-relativistic quantum mechanics with a δ function potential in the limit $c\to\infty$.

I. Introduction

In this paper we are concerned with the general question of how relativistic quantum mechanics with speed of light c is approximated by non-relativistic quantum mechanics in the limit $c\to\infty$. Only a few rigorous results of this nature exist. For example, for a single particle in an external field, the relation between the Dirac equation and the Schrödinger equation is understood. ([12], and earlier references.)

Specifically we consider $\mathscr{P}(\varphi)_2$ quantum field theory models with speed of light c, denoted $\mathscr{P}(\varphi)_{2,c}$. According to the folklore the $c\to\infty$ limit should produce a multiparticle Schrödinger theory with δ-function potentials. For $(\varphi^4)_{2,c}$ the argument goes as follows. Set

$$\omega_c(p)=(p^2c^2+m^2c^4)^{1/2} \qquad p\in\mathbb{R}^1$$

$$\varphi_c(x)=(2\pi)^{-1/2}\int e^{-ipx}c(2\omega_c(p))^{-1/2}(a^*(p)+a(-p))dp\ ,$$

where m is the single particle mass and a^*, a are the usual creation and annihilation operators. The Hamiltonian for the theory has the form

$$H_c=\int a^*(p)\omega_c(p)a(p)dp+\lambda\int :\varphi_c^4(x):dx\ .$$

As $c\to\infty$ all creation and annihilation processes are somehow kinematically suppressed. If we also ignore the "zitterbewegung" term mc^2 in $\omega_c(p)=mc^2+(2m)^{-1}p^2+\mathcal{O}(c^{-2})$, then in some vague sense we have

$$H_\infty=\int a^*(p)(2m)^{-1}p^2a(p)dp + \frac{1}{2}\left(\frac{3\lambda}{m^2}\right)\int a^*(x)a^*(y)\delta(x-y)a(x)a(y)dxdy\ .$$

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