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Pressure and Variational Principle for Random Ising Model

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Abstract. An Ising model traditionally is a model for a repartition of spins on a lattice. Griffiths and Lebowitz ([3. 5]) have considered distributions of spins which can occur only on some randomly prescribed sites—Edwards and Anderson have introduced models where the interaction was random ([6, 7]). In both cases, the formalism of statistical mechanics reduces mainly to a relativised variational principle, which has been proved recently by Walters and the author [1]. In this note, we show how that reduction works and formulate the corresponding results on an example of either model.

1. Notations and Results

Let $Y=\{0,1\}^{\boldsymbol{Z}^d}$, $X=\{0,+1,-1\}^{\boldsymbol{Z}^d}$ be the sets of configurations of particles (respectively of particles with a spin) on a lattice $\boldsymbol{Z}^d$, Let $\pi:X\to Y$ denote the natural map such that $(\pi(x))_s=|x_s|$ for s in $\boldsymbol{Z}^d$, τ_s the shift transformations on X and Y, Λ_n the positive cube of side n containing the point $(0,0,\dots,0)$ of $\boldsymbol{Z}^d$. A point y is said generic for an invariant measure ν on Y if the measures $\frac{1}{n^d}\sum_{s\in\Lambda_n}\delta_{\tau_s y}$ converge towards the measure ν (δ_z denotes the Dirac measure at the point z).

Let J,h be real numbers. For x in X with $x_s=0$ except for a finite number of s, define:

$$U(x)=\sum_{s\in\boldsymbol{Z}^d}hx_s+\sum_{\substack{s,t\in\boldsymbol{Z}^d\\|s-t|=1}}Jx_sx_t\,,$$

where $|s|=\sum_i|s_i|$ if $s=(s_i,\,i=1,\dots,d)$.

For any finite subset Λ of $\boldsymbol{Z}^d$ and any y in Y let us consider the partition function of the box Λ above $yZ_\Lambda(y)$:

$$Z_\Lambda(y)=\sum\exp(-U(x))\,,$$

where the summation is made over the set of x such that $|x_s|=y_s$ for s in Λ, $x_s=0$ elsewhere. Let $M(X,\tau)$ denote the set of invariant probability measures on X.