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# The Energy-Momentum Spectrum in the $P(\varphi)_2$ Quantum Field Theory

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**Abstract.** The $P(\varphi)_2$ interaction with the periodic boundary conditions is considered. It is shown that the energy-momentum spectrum lies in the forward light cone. As a consequence, this result implies that the $P(\varphi)_2$ theory in the infinite volume with the periodic boundary conditions is Lorentz invariant.

## 1. Introduction

In the present paper we prove the spectral condition for models with the $P(\varphi)_2$ interaction with the periodic boundary conditions.

The paper [1] by Glimm and Jaffe was devoted to the investigation of the energy-momentum spectrum. Their arguments were based on the uniqueness of the vacuum for the Lorentz rotated Hamiltonian $H_V(\beta)\,(=\beta_0 H_V+\beta P_V)$ and on the fact that the uniqueness of the vacuum and the translation invariance of the Hamiltonian imply the translation invariance of the vacuum, that is, the vacuum has the zero momentum.

Unfortunately, the semigroup of operators $\exp(-tH_V(\beta))$ does not preserve the positivity even for the free Hamiltonian and so the uniqueness proof which is based on the positivity preservation and the hypercontractivity does not work.

In our proof we do not prove the uniqueness of the vacuum, instead of this we use, following Seiler and Simon [2], the Osterwalder-Schrader positivity in the spatial direction to show that the vacuum subspace of $H_V(\beta)$ contains the vector with the zero momentum. Thus, our arguments may be represented by the following diagram.