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The Application of Dewitt-Morette Path Integrals to General Relativity

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Abstract. The formulation of path integrals in terms of pseudomeasures by Cecile DeWitt-Morette is extended to infinite-dimensional state-spaces and to the state spaces dual to nuclear spaces appropriate to second-quantisation. In both cases a "distribution" formulation is given to allow a subsequent extension to manifolds. It is shown that the resulting theory is "correct" in that it can give rise to a wave function on state space which obeys a Schrödinger equation in appropriate circumstances. The corresponding state manifolds for quantum gravity are then defined, and the conditions under which the theory extends to them are discussed. It is shown in an appendix that the Riemannian metric required by the theory exists on one of the types of state manifold for a wide class of cases.

1. Introduction and Synopsis

(a) The Idea of Path Integrals

We consider a dynamical system whose state at time τ is represented by a point $q(\tau)$ in a configuration space E. Thus as τ varies from 0 to t, $q(\tau)$ can describe a *path* $q:[0,t]=T\to E$. Given an initial state $O=q(0)\in E$, we examine the set Φ of all C^∞ paths starting at O.

The basic idea of the path-integral formalism is to quantise the system by defining a wave-function $\psi(x)$ for the state at time t by the formula

$$\int_A d\Psi(x) \overset{\text{def}}{=} \int_A \psi(x)d\mu(x) = \int_{\Phi_1} \chi_A(q(t))d\nu(q) \tag{1}$$

for any $A\subset E$. Here μ is some "standard" measure on E, χ_A is the characteristic function of A. Φ has been completed in a suitable metric to Φ_1, and ν is a specially