Commun. math. Phys. 56, 125—146 (1977)

Communications in Mathematical Physics

The Vlasov Dynamics and Its Fluctuations in the 1/N Limit of Interacting Classical Particles

W. Braun and K. Hepp
Physics Department, ETH, CH-8093 Zürich, Switzerland

Abstract. For classical N-particle systems with pair interaction $N^{-1} \sum_{1 \leqq i \leqq j \leqq N} \phi(\underline{q}_i - \underline{q}_j)$ the Vlasov dynamics is shown to be the w^*-limit as $N \to \infty$. Propagation of molecular chaos holds in this limit, and the fluctuations of intensive observables converge to a Gaussian stochastic process.

§ 1. Introduction

Consider the Newtonian equation

$$\ddot{\underline{x}}(t, a, \mu) = \int \mu(db) \underline{F}(\underline{x}(t, a, \mu) - \underline{x}(t, b, \mu)) \tag{1.1}$$

for a particle with initial condition

$$z(0, a, \mu) = (\underline{x}(0, a, \mu), \dot{\underline{x}}(0, a, \mu)) = a = (\underline{q}, \underline{p}) \tag{1.2}$$

interacting via a regular 2-body force $\underline{F}(\underline{q}) = -\underline{V}\phi(\underline{q}) = -\underline{F}(-\underline{q})$ with other particles having initial conditions distributed over a real Borel measure μ on $\mathbb{R}^6$. This framework contains the canonical dynamics of N mass points

$$\ddot{\underline{x}}_n(t, \alpha_N) = \sum_{m=1}^{N} \underline{F}(\underline{x}_n(t, \alpha_N) - \underline{x}_m(t, \alpha_N)), \tag{1.3}$$

where $1 \leqq n \leqq N$ and with initial condition $\alpha_N = (a_1, \ldots, a_N)$. For, let $\mu^{\alpha_N}(da) = \sum_n \delta_{a_n}(da)$ and $\underline{x}(t, a, \mu^{\alpha_N})$ be the solution of (1.1). Then

$$\underline{x}_n(t, \alpha_N) = \underline{x}(t, a_n, \mu^{\alpha_N}) \tag{1.4}$$

is the solution of (1.3). On the other hand for $\mu^f(da) = f(a)da$ the Newtonian Equation (1.1) also solves the Vlasov Equation [1]:

$$\frac{\partial f}{\partial t}(t, a) = -\underline{p} \frac{\partial f}{\partial \underline{q}}(t, a) - \frac{\partial f}{\partial \underline{p}}(t, a) \int da' f(t, a') \underline{F}(\underline{q} - \underline{q}') \tag{1.5}$$