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On Symmetric Gauge Fields

A. S. Schwarz

Moscow Physical Engineering Institute, Moscow M 409, USSR

Abstract. The subgroups of the symmetry group of the gauge invariant Lagrangian are studied. For given subgroup G the G-invariant gauge fields are listed.

Let $F(\varphi)$ be a G-invariant functional and let H be a subgroup of the symmetry group G. It is easy to prove under certain conditions that every extremal of the functional $F(\varphi)$ considered only in the H-invariant fields is an extremal of this functional on all fields (see for instance [1]). This assertion can be used to search solutions of classical field equations especially in gauge theories. In these theories the functionals under consideration are invariant with respect to the group R generated by local gauge transformations and spatial symmetries. To apply the assertion above one must find the subgroups of the group R and for given subgroup $G \subset R$ one must find all G-invariant fields. In present paper we solve these two problems. Some results in this direction were obtained earlier by Burlankov [2] and used in [9].

To facilitate the reading to physicists we have divided the paper in two sections. The considerations of Section 1 used only notions familiar to physicists but in Section 2 we use the geometrical language of fibre space theory (see for instance [3]).

All manifolds and all maps under consideration will be supposed smooth.

Section 1

We denote by O the group of spatial symmetries. (This group acts on a manifold M; in physical applications usually M is three-dimensional or four-dimensional euclidean space.) The group of local gauge transformations will be denoted by K_∞ and the group generated by K_∞ and O will be denoted by R. The group K_∞ can be identified with the group of smooth functions on M taking values in the gauge group K. The group R can be considered as the group of pairs $(k(x), g)$ where $k(x) \in K_\infty$, $g \in G$ and the product of pairs $(k_1(x), g_1) \in R$, $(k_2(x), g_2) \in R$ is a pair $(k(x), g)$