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Geometric Methods in Multiparticle Quantum Systems

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Abstract. Technically simple proofs are given of the HVZ theorem on the bottom of the essential spectrum of multiparticle systems and of Combes' result on completeness below the lowest three body threshold. The first proof is a variant of a proof of Enss and a decendent of Zhislin's original proof. Finally, we apply our methods to the bound state spectrum.

§ 1. Introduction

This is the second of a series of papers that attempts to develop the basic spectral and scattering properties of multiparticle nonrelativistic Schrödinger operators without the use of resolvent equations. The first paper in the series, written jointly with P. Deift, [15], showed how to reduce the completeness of the scattering theory to the existence of certain time-dependent operators (which remain to be controlled). In that paper an important element was the idea of using the geometry of configuration space to separate channels. This idea is basic to the present paper; indeed it is the central character in the drama with various technical results playing merely supporting roles.

My paper with Deift was not the first one to suggest the use of time-dependent methods in studying the completeness of multichannel multiparticle systems. In 1967, J. M. Combes [9] published an extremely deep paper on completeness of N-particle systems below the lowest energy necessary for breakup into three or more clusters. This paper has been largely ignored, probably in part because it is technically somewhat complex. My original motivation in the present work was a desire to use the insights of [15] to find a simpler proof of Combes' result. I expected (correctly so) that the idea of Pearson [29] of systematically exploiting the "two Hilbert space" operator

$$\Omega^{\pm}(A,B;J) = \underset{t\to\mp\infty}{s\text{-lim}}\, e^{iAt} J\, e^{-iBt} E_{\mathrm{ac}}(B) \tag{1.1}$$

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